Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 10: Absolute Value

Definition 1. **Absolute Value** *Geometrically, the absolute value of a real number a is the distance from the origin on the number line to the point with coordinate a. The definition of absolute value is*

$$|a| = a \quad \text{if} \quad a \geq 0 \qquad \text{and} \quad |a| = -a \quad \text{if} \quad a < 0.$$

For example, $|-5| = -(-5) = 5$ and $|5| = 5$. This means that the **absolute value of any real number is greater or equal to zero**.

Definition 2. **Rules for Solving Absolute Value Equations:**
If $a > 0$ and u is an algebraic expression, then

$$|u| = a \quad \text{is equivalent to} \quad u = a \text{ or } u = -a$$

$$|u| = -a \quad \text{has no solution.}$$

Note that if $|u| = 0$, then $u = 0$ is the only solution.

Example 1. *Solve:* $|2x - 3| - 5 = 8$
Solution:

$$\begin{aligned} |2x - 3| - 5 &= 8 \\ |2x - 3| &= 13 \qquad \text{add 5 to both sides and simplify} \end{aligned}$$

Then applying the rules, either $2x - 3 = 13$ or $2x - 3 = -13$. So,

$$\begin{aligned} 2x - 3 = 13 \qquad &\text{or} \qquad 2x - 3 = -13 \\ 2x = 16 \qquad &\text{or} \qquad 2x = -10 \\ x = 8 \qquad &\text{or} \qquad x = -5 \end{aligned}$$

After checking the answers in the original equation, then the solution set is $\{-5, 8\}$.

Example 2. *Solve:* $|x - 1| = |x + 5|$
Solution: *To solve $|x - 1| = |x + 5|$, we need to solve $x - 1 = x + 5$ or $x - 1 = -(x + 5)$. So,*

$$\begin{aligned} x - 1 = x + 5 \qquad &\text{or} \qquad x - 1 = -(x + 5) \\ -1 = 5 \text{ False} \qquad &\text{or} \qquad x = -2 \end{aligned}$$

Hence, the solution set is $\{-2\}$.

Definition 3. **Rules for Solving Absolute Value Inequalities:**
If $a > 0$ and u is an algebraic expression, then

$$|u| < a \qquad \text{is equivalent to} \qquad -a < u < a.$$

$$|u| \leq a \qquad \text{is equivalent to} \qquad -a \leq u \leq a.$$

$$|u| > a \qquad \text{is equivalent to} \qquad u < -a \text{ or} u > a.$$

$$|u| \geq a \qquad \text{is equivalent to} \qquad u \leq -a \text{ or} u \geq a.$$

Example 3. *Solve:* $|4x-1| \leq 9$
Solution: $|4x-1| \leq 9$ *is equivalent to* $-9 \leq 4x-1 \leq 9$*. So,*

$$\begin{aligned} -9 &\leq 4x-1 \leq 9 \\ 1-9 &\leq 4x-1+1 \leq 9+1 \\ -8 &\leq 4x \leq 10 \\ \frac{-8}{4} &\leq \frac{4x}{4} \leq \frac{10}{4} \\ -2 &\leq x \leq \frac{5}{2} \end{aligned}$$

So the solution set is $[-2, \frac{5}{2}]$.

Example 4. *Solve:* $|2x-8| > 4$
Solution: $|2x-8| > 8$ *is equivalent to* $2x-8 < -4$ *or* $2x-8 > 4$*. So,*

$$\begin{aligned} 2x-8 < -4 \quad &\textit{or} \quad 2x-8 > 4 \\ 2x-8+8 < -4+8 \quad &\textit{or} \quad 2x-8+8 > 4+8 \\ 2x < 4 \quad &\textit{or} \quad 2x > 12 \\ x < 2 \quad &\textit{or} \quad x > 6 \end{aligned}$$

Hence, the solution set is $(-\infty, 2) \cup (6, \infty)$*. Recall that* ($or = \cup$) *and* ($and = \cap$).

Remark 1. *The inequality* $|3x-2| \leq -5$ *has no solution because the absolute value is always nonnegative.*
The inequality $|5x+3| > -2$ *is true for all real numbers because the absolute value is always nonnegative. Hence the solution set of such an inequality is* $(-\infty, \infty)$.